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Transitory Exogenous Shocks in a Non-Linear Framework: Application to the Cyclical Behaviour of the German Aggregate Wage Earnings

Mohamed Chikhi & Claude Diebolt *

Abstract: *»Transitorische exogene Schocks in einem nicht-linearen Rahmen: Anwendung auf das zyklische Verhalten des aggregierten deutschen Lohneinkommens«*. This paper analyses the cyclical behaviour of the German annual aggregate wage earnings over 179 years. Our results show that there are transitory exogenous shocks which contain predictive information for aggregate wages in a non-linear framework.

Keywords: Time Series Econometrics, Wages, Germany.

This paper analyses, in extension to a previous publication by Diebolt (2008), the cyclical behavior of the German annual aggregate wage earnings over 179 years (see Figure 1). The Dickey-Fuller and Philips-Perron unit-root tests show that one unit root exists in this series (see Table 1). The Schmidt-Phillips unit root test is applied in order to confirm these results. These tests are particularly important because they are robust with the heteroscedasticity. Our time series is finally log-differenced to give the stationary aggregate wages earnings series in Figure 2. As shown in Table 2, there is strong evidence that the null hypothesis of normality is rejected. Leptokurtosis in the data is revealed by a high Kurtosis coefficient of 24.77 and a Jarque-Bera statistic of 3539.18. Moreover, the Cramer-Von Mises, Watson and Anderson-Darling statistics are greater than the critical value at significance level 5%. The rejection of normality is confirmed. The nonparametric kernel density estimator confirms these results. Figure 3 shows the asymmetry and leptokurtic character of the distribution. This asymmetry may be caused by the presence of nonlinearities in this series. Moreover, the scatter diagram (see Figure 4) confirms these results and shows that the obtained form is irregular ellipsoid and the distribution is typically non-Gaussian. As can be seen in Table 3, we note that the random walk hypothesis is clearly rejected. The nonparametric BDS statistics, which test for the pres-

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ence of a linear or nonlinear dependence, are greater than the critical value at 5%. Therefore, the aggregate wages earnings process is mixing and predictable because this test led us to reject the null *iid* hypothesis. This test generally shows the presence of a significant autocorrelations different to zero in the short term but it is impossible to exploit these autocorrelations to identify the characters of shocks in the aggregate wages.

Figure 1: Aggregate Wages Earnings series *W*

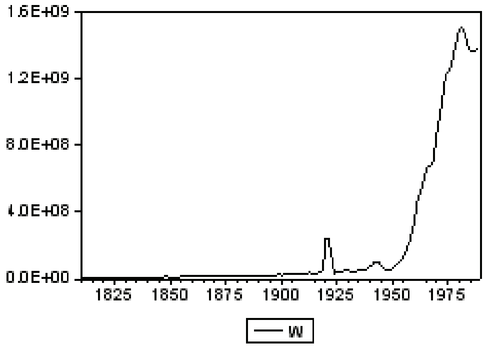


Table 1: Dickey-Fuller, Philips-Perron and Schmidt-Philips Unit Root Tests

Testing procedure	Level		First difference	
Dickey-Fuller test statistic	Model 3	-2.2880* (-3.4360)	Model 2	-7.4950* (-1.9416)
Philips-Perron test statistic	Model 3	-2.2039* (-3.4357)	Model 1	-8.8715* (-1.9415)
Schmidt-Philips test statistic	*Z(Rho)*	-9.5934* (-18.1)	*Z(Rho)*	-113.24* (-18.1)
	Z(Tau)	-2.2055* (-3.02)	*Z(Tau)*	-9.0971* (-3.02)

Model 1: model with intercept

Model 3: model with intercept and deterministic trend

Model 2: model –none

* Critical values for rejection of hypothesis of a unit root at significance level 5%.

Figure 2: Aggregate Wages Earnings series in logarithm (in level and first difference)

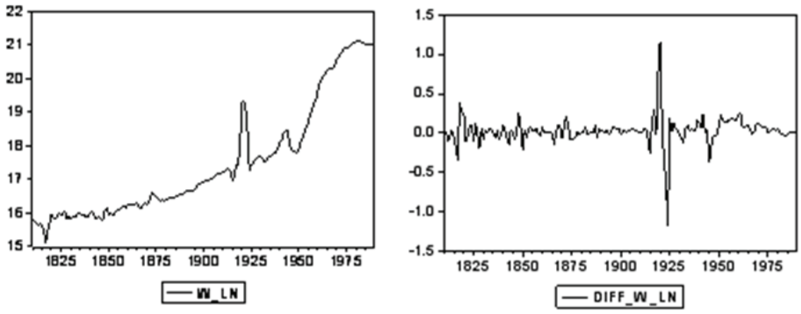
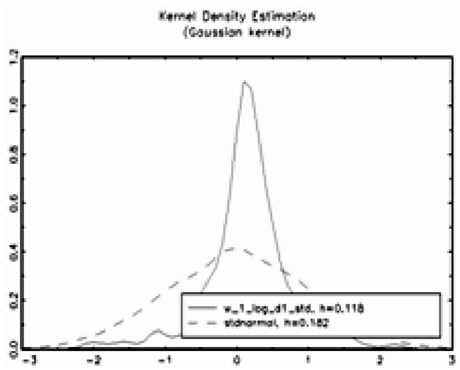


Table 2: The normality tests for log-differenced aggregate wage earnings series

Series	Skewness	Kurtosis	J.B Statistic	Cramer-Von Mises stat.	Watson stat.	Anderson-Darling Stat.
DIFF_W_LN	-0.3415	24.7729	3539.184	2.3776	2.376593	12.77288

Figure 3: The nonparametric kernel density estimator

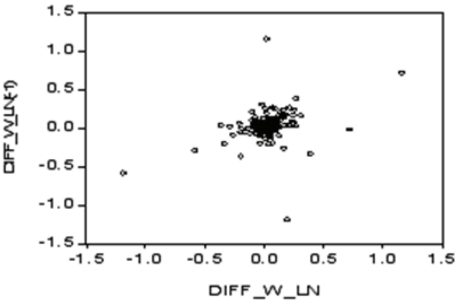


Note: We apologize for the insufficient quality of several figures. Unfortunately, we were not able to reconstruct the original data/ figures for technical reasons.

Table 3: The BDS independence test for log-differenced aggregate wage earnings series

m	Fraction of pairs	Standard deviations
2	6.309390	6.755904
3	7.472316	8.232013
4	8.029945	9.218113
5	8.260612	9.917872
6	8.756026	11.03920
7	9.489858	12.54947
8	10.33077	14.51925
9	11.31220	17.04303
10	12.32023	20.01797
11	13.52798	23.70146
12	14.72975	27.93491
13	15.97084	32.82012
14	17.31572	38.57481
15	18.72403	45.00404
16	20.48078	52.76547
17	22.81037	65.09565
18	25.48341	80.23012
19	28.61621	97.99633
20	32.24208	119.9214

Figure 4: Scatter Diagram Attractor



We test this hypothesis while working on longer horizons. The nonparametric BDS test led us to reject the null *iid* hypothesis but it cannot detect the presence of a long term dependence structure. For this reason, we retain in this application to the aggregate wages series two values for the ordinates of the periodogram in order to frame the squared root number of observations. As is seen in Table 4, there is strong evidence that the aggregate wages earnings series exhibits short memory. The statistic values are smaller than the critical value at 5%. Nevertheless, the aggregate wages are predictable in the short term. So, the observed aggregate wages movements appear to be the result of transitory exogenous shocks, i.e. the consequence of the shocks are therefore transitory.

Table 4: The semi-parametric estimates of long memory coefficient for log-differenced aggregate wage earnings series

Bandwidth	Ordinates	
	$T^{0.5}$	$T^{0.8}$
GPH	0.1521 (0.6281)	-0.0251 (-0.2716)
Rectangular.	0.0445 (0.1668)	-0.0147 (-0.1406)
Bartlett	0.1030 (0.6681)	-0.0313 (-0.5315)
Daniell	0.0982 (0.5200)	-0.0311 (-0.4320)
Tukey	0.0894 (0.5310)	-0.0271 (-0.4218)
Parzen	0.1159 (0.8357)	-0.0309 (-0.5832)
B-priest	0.0667 (0.3228)	-0.0218 (-0.2761)
Andrews-Guggenberger	0.0211 (0.2418)	-

In order to capture the characteristics of shocks, the modeling of aggregate wages turns towards a nonlinear framework because the economies may have differentiated dynamics and macroeconomic variables may have different effects on aggregate wages depending on the business cycle regime. We use here the smooth transition autoregressive model (STAR)[1] introduced by Teräsvirta (1994).

Our modeling technique is based on the grid search procedures for the selection of the appropriate transition variable (see Öcal and Osborn, 2000). The first stage in the modeling cycle is to test linearity against STAR by selecting a linear model with residuals, which are approximately white noise and start with an order of 6 lags. The lag order of the model could be determined by an order

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[1] The STAR model has the form: $y_t = \phi' z_t + \theta' z_t G(\gamma, c, s_t) + \varepsilon_t$, $\varepsilon_t \sim iid(0, \sigma^2)$ with: the transition function

$$G(\gamma, c, s_t) = \left(1 + \exp\left(-\gamma \prod_{k=1}^{K} (s_t - c_k)\right)\right)^{-1}, \ \gamma > 0$$

where: $z_t = (1, y_{t-1}, ..., y_{t-p})'$ ϕ and θ are the parameter vectors of the linear and the nonlinear part respectively. The parameter s_t can be part of z_t or it can be another variable, like for example a deterministic trend.

selection criterion such as the Akaike criterion (AIC). The selected linear model obtained by the general-to-specific procedure and based on the AIC is assumed to form the null hypothesis for testing linearity. The test, to be referred as the particular LSTAR linearity test in our work is carried out for different candidate transition variables. It can be obtained from the following auxiliary regression (see Luukkonen, Saikkonen and Teräsvirta, 1988):

$$y_t = \beta_0^{'} z_t + \sum_{j=1}^{3} \beta_j^{'} \hat{z}_t s_t^j + \varepsilon_t^*$$

with $z = (1, \hat{z}_t)^{'}$. The null hypothesis of linearity is $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ and we compute the statistic in this equation for several candidate transition variables and the one for which the p-value of the test is smallest (strongest rejection of linearity) is selected as the true transition variable.

Once linearity is rejected against STAR, the second stage is to select the appropriate transition variable and proceed to estimate the parameters of the model. For each candidate transition variable, a two-dimensional grid search is carried out using at least 200 values of γ (1 to 200 with the range extended if the minimizing value of γ is close to 200) and 20 equally spaced values of c within the observed range of the transition variable. Essentially, the transition variable series is ordered by value. The model with the minimum SSR value from the grid search procedure is used to provide the γ, c and s for an initial estimate of the transition function. After, we use the non-linear least squares (NLS) framework for reducing the order of the model. Giving fixed values to the parameters of the transition function makes the STAR model linear in the remaining coefficients. The grid search mentioned above is used to obtain sensible initial values. Conditional on this transition function, the parameters of the STAR model can be estimated by OLS and we call this model the linearised version of the STAR model. To determine the order of the linear STAR we follow a general-to-specific procedure and the selected model is based on the AIC criterion. The estimated coefficients from the linear STAR along with the transition function parameters from the grid search are used as initial values in the non-linear estimation in the next stage. The preferred model is re-estimated (including the transition function parameters) by NLS using the BHHH algorithm. After estimating the parameters of the STAR, these are compared with those obtained from the linearised version since the latter is used for model specification.

The empirical results show that the AIC obtained by the general-to-specific methodology is minimized when DIFF_W_LN_1, DIFF_W_LN_2 and DIFF_W_LN_3 are considered as the selected variables. So, we test linearity against general and particular non-linearity. The linearity tests are displayed in

Table 5, while Table 6 reports the grid search results. Note that the p-value (0.011) of the LST test indicates that linearity can be rejected. In particular linearity tests, the null is rejected in two out of three cases. The strongest evidence of LSTAR non-linearity occurs when DIFF_W_LN_3 is used as the transition variable. Admittedly, the statistical evidence of non-linearity is quite strong. The grid search results, however, show that SSR is minimized when DIFF_W_LN_1 is considered as the switching variable (see also Figure 5).

Table 5: The linearity tests

General linear		Particular linear	
RESET	LST	Transition variable	LSTAR
0.267	0.011*	DIFF_W_LN_1	0.116
		DIFF_W_LN_2	0.023*
		DIFF_W_LN_3	0.008*

Table 6: Grid search results for the specification of the STAR model

s_t	γ	c	SSR
DIFF_W_LN_1	64	-0.2969	2.8495
DIFF_W_LN_2	3	-0.2697	4.1792
DIFF_W_LN_3	2	-0.1078	3.5176

Figure 5: Graphical representation of grid search for start values

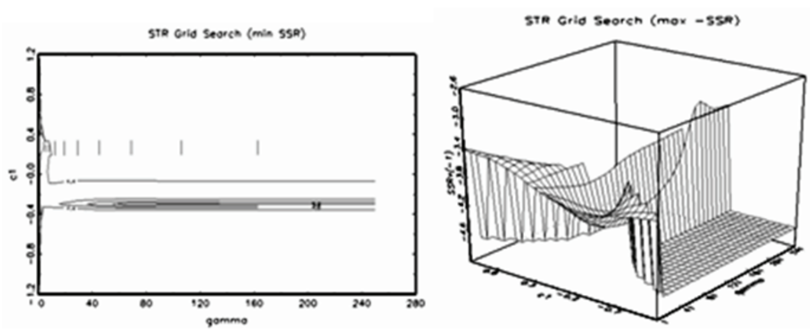


Table 7 reports the estimation results of the STAR model (see also Figure 6) with the linear and nonlinear parts (see also Figure 7). We note that the all parameters are significant; the student statistics are greater than the critical value at significance level 5% and the residuals shown in Figure 8 are characterized by the white noise process and don't present any serial correlation (see Breusch-Godfrey statistic with two lags) and any conditional heteroscedasticity as $1.6523 < \chi^2(1)$. Furthermore, the normality hypothesis is accepted, the residuals are characterized by a Gaussian distribution because $JB = 0.9733 < \chi^2(2)$. Figure 9 displays the transition function. The estimated slope parameter values imply very different dynamics around the threshold parameters. According to our model, the large value of the slope parameter implies almost instantaneous switch. This can also be seen in the second panel of Figure 9, where the transition function fluctuates only between zero and one; there are no intermediate values.

Table 7: Estimation of STAR model by NLS using BHHH algorithm

Variable	Linear part	Nonlinear part
Constant	0.0337 (4.1723)	-0.1622 (-4.0560)
DIFF_W_LN_1	0.1574 (2.8551)	-
DIFF_W_LN_2	-0.3569 (-3.0244)	-
DIFF_W_LN_3	-	-0.1587 (-2.0862)
s_t	DIFF_W_LN_1	
γ	64.3769	
c	-0.2969	
AIC /SC / HQ	-4.0098 / -3.8297 / -3.9367	
R^2 / Adjusted R^2	0.4879 / 0.4909	
Skewness	-0.1832	
Kurtosis	2.0431	
Normality	0.9733	
ARCH(1)	1.6523	
Breusch-Godfrey stat.	3.3207	

Figure 6: Original and fitted aggregate wages series in logarithm (first-difference)

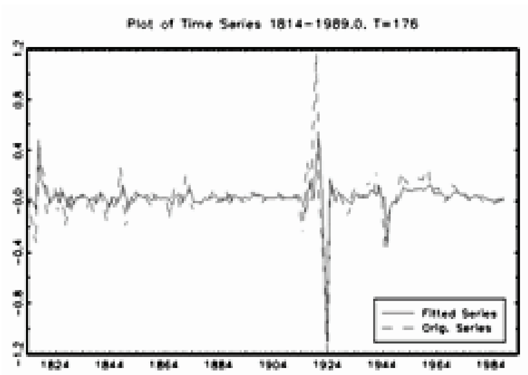


Figure 7: Linear and nonlinear part of STAR model

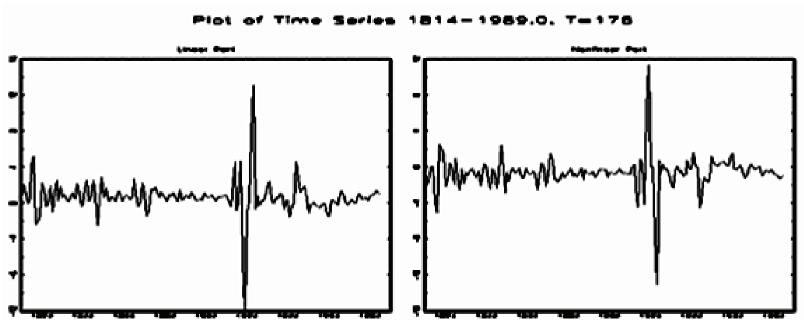


Figure 8: Standardized residuals of STAR models

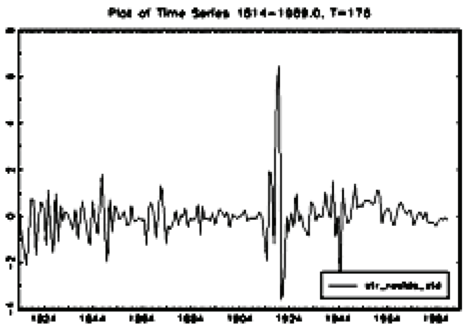
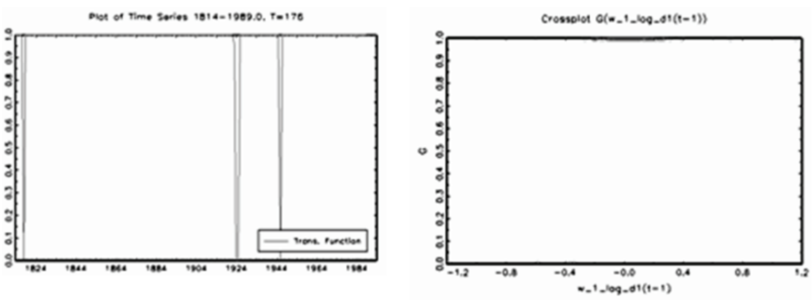


Figure 9: Transition function of LSTAR model versus diff_ln_w_1 (upper panel) and over time (lower panel)



The STAR model was re-estimated until 1979 and the 10 subsequent date sets were forecast. In this study, the predictive qualities of the STAR process are compared with the linear model and the random operation model. The forecasting results are reported in Table 8. It is noted that the forecasts provided by the linear and STAR models are better than the forecast resulting from a random operation model. On the other hand, according to the 1980-1989 forecast period results, the corresponding non-linear version beats the linear model. This nonlinear model provides the smallest Mean Quadratic Error (MQE) and Mean Absolute Error (MAE) value. So, it is striking the bad performance of the random walk model in terms of MQE and MAE criteria. These results have shown that there are transitory exogenous shocks which contain predictive information for aggregate wages in a non-linear framework. Consequently, the aggregate wages series becomes predictable in short term because the nature of shocks is neither permanent nor durable. Indeed, the observed movements appear to be the result of transitory exogenous shocks.

Table 8: Forecast performance

Criterion	Linear	LSTAR	Random Walk
MQE	0.0348	0.0322	0.0354
MAE	0.0213	0.0204	0.0219

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